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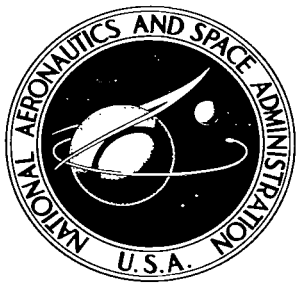

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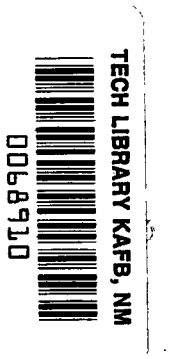

PARTICLE-LIKE SOLUTIONS TAKING INTO ACCOUNT GRAVITATION

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ABSTRACT

The effect of gravitation on particle-like solutions is investigated for the case of a scalar complex potential when the gravitational field is produced only by the mass of a spherically symmetric scalar potential. It is shown that the gravitational interaction is negligible if the ratio of the gravitational radius to the radius of the particle is much less than unity. Thus for a particle with a radius of $r_0 \sim 10^{-13}$ cm and a mass of $\mu \sim 10^{-27}$ gm, the gravitational correction is approximately 10^{-40} percent.

Particle-like solutions in nonlinear classical field theory are convenient because the field potential, the tensor of the energy-impulse, has no singularities. In addition, the field energy is concentrated in a small region of space in a sphere whose radius is comparable to the radius of an elementary particle.

For the Lagrangian

$$L = -\frac{\partial \psi^*}{\partial x_l}\frac{\partial \psi}{\partial x_l} - m_0^2 [\psi^*\psi + F(\psi^*\psi)] \qquad (1)$$

of the scalar potential ψ (F is the nonlinear part of the Lagrangian) the particle-like solutions have been investigated in considerable detail (refs. 1-7).

If we let (ref. 1)

$$\psi = \varphi e^{i\varepsilon x_4}, \tag{2}$$

the Lagrange equation takes the form (in the case of spherical symmetry)

$$\frac{\partial^2\varphi}{\partial r^2} + \frac{2}{r}\frac{\partial\varphi}{\partial r} - (m_0^2 - \varepsilon^2)\varphi - m_0^2\frac{\partial F}{\partial\varphi} = 0. \tag{3}$$

The most complete investigation has been made of the solution with step-like nonlinearity for which

$$F(\varphi) = \begin{array}{ll} 0 & \text{when } |\varphi| < A \\ -\lambda\varphi & \text{when } |\varphi| > A \end{array}$$

and with cubic nonlinearity where $F(\varphi) = -\frac{\lambda}{2}\varphi^4$.

In these cases the particle-like solutions have a character similar to the variation of the wave function for a particle in a potential hole of finite depth. Specifically: $\varphi \sim \frac{1}{r}\sin\sqrt{(\varepsilon^2 - m_0^2 + \lambda m_0^2)}$ within the particle radius and $\varphi \sim \frac{1}{r}e^{-\sqrt{m_0^2-\varepsilon^2}\,r}$ outside the particle radius.

The investigations mentioned above did not take into account, in explicit form, the distortion of the metrics in a particle due to the field mass potential and therefore, strictly speaking, the field equation should be written in the general covariant form, particularly since M. F. Shirokov (ref. 9) has pointed out that in the Bopp-Infeld and the Bopp-Podol'skiy theories the distortion of the metrics cannot be neglected since in the concept of the metrics tensor $g_{ii} = 1 + h_{ii}$, either h_{ii} (Born-Infeld theory) or h_{44} (Bopp-Podol'skiy) diverges at zero because of the singularity of the field at the center of the particle.

Since the particle-like solutions are regular everywhere, the designations of M. F. Shirokov cannot be applied directly to the present case and the gravitational corrections must be investigated separately.

The Lagrangian function, analogous to (1) may be easily reduced to a covariant form. Indeed

$$L = -\frac{\partial\psi^*}{\partial x^l}g^{lk}\frac{\partial\psi}{\partial x^k} - m_0^2[\psi^*\psi + F(\psi^*\psi)]. \tag{4}$$

The tensor of the energy-impulse, given by the equation (ref. 10)

$$T_i^k = \frac{2g^{km}}{\sqrt{-g}}\left[\frac{\partial}{\partial x^e}\frac{\partial(\sqrt{-g}\,L)}{\partial\left(\frac{\partial g^{im}}{\partial x^e}\right)} - \frac{\partial(\sqrt{-g}\,L)}{\partial g^{im}}\right],$$

for the curvature coordinates (ref. 11) with a spherically symmetric potential φ, has the following components which differ from zero:

$$T_1^1 = e^{-\lambda}\left(\frac{\partial\varphi}{\partial r}\right)^2 + \varepsilon^2 e^{-\nu}\varphi^2 - m_0^2(\varphi^2 + F),$$

$$T_2^2 = T_3^3 = -e^{-\lambda}\left(\frac{\partial\varphi}{\partial r}\right)^2 + \varepsilon^2 e^{-\nu}\varphi^2 - m_0^2(\varphi^2 + F),$$

$$T_4^4 = -e^{-\lambda}\left(\frac{\partial\varphi}{\partial r}\right)^2 - \varepsilon^2 e^{-\nu}\varphi^2 - m_0^2(\varphi^2 + F),$$

$$(e^{\lambda} = g_{11}(r);\ -e^{\nu} = g_{44}(r)).$$

The linearly independent components of the Einstein equation $R_i^k - \frac{1}{2}R\delta_i^k = \frac{8\pi k}{c^4}T_i^k$ have the form

$$e^{-\lambda}\left(\frac{1}{r^2} - \frac{\frac{\partial\lambda}{\partial r}}{r}\right) - \frac{1}{r^2} = -\frac{8\pi k}{r^4}\left[e^{-\lambda}\left(\frac{\partial\varphi}{\partial r}\right)^2 + \varepsilon^2 e^{-\nu}\varphi^2 + m_0^2(\varphi^2 + F)\right],$$

$$e^{-\lambda}\left(\frac{1}{r^2} + \frac{\frac{\partial\nu}{\partial r}}{r}\right) - \frac{1}{r^2} = \frac{8\pi k}{r^4}\left[e^{-\lambda}\left(\frac{\partial\varphi}{\partial r}\right)^2 + \varepsilon^2 e^{-\nu}\varphi^2 - m_0^2(\varphi^2 + F)\right].$$

In order that the system of equations which determines the three functions φ, λ, ν as a function of the argument r be complete, we add the Lagrange equation to the Einstein equation

$$\frac{\partial}{\partial x^i}\left[\frac{\partial(\sqrt{-g}\,L)}{\partial\frac{\partial\psi^*}{\partial x^i}}\right] - \frac{\partial(\sqrt{-g}L)}{\partial\psi^*} = 0,$$

After a direct calculation this leads to

$$e^{-\lambda}\left[\frac{\partial^2\varphi}{\partial r^2} + \left(\frac{2}{r} + \frac{\frac{\partial\nu}{\partial r} - \frac{\partial\lambda}{\partial r}}{2}\right)\frac{\partial\varphi}{\partial r}\right] + (\varepsilon^2 e^{-\nu} - m_0^2)\varphi - m_0^2\frac{\partial F}{\partial\varphi} = 0.$$

In particular, for a step-like nonlinearity we obtain a system of equations

$$e^{-\lambda}\left(\frac{1}{r^2}-\frac{\lambda'}{r}\right)-\frac{1}{r^2}=-\frac{8\pi k}{c^4}\left[e^{-\lambda}\varphi'^2+(\varepsilon^2e^{-\nu}+m_k^2)\varphi^2\right], \tag{5}$$

$$e^{-\lambda}\left(\frac{1}{r^2}+\frac{\nu'}{r}\right)-\frac{1}{r^2}=\frac{8\pi k}{c^4}\left[e^{-\lambda}\varphi'^2+(\varepsilon^2e^{-\nu}-m_k^2)\varphi^2\right], \tag{6}$$

$$e^{-\lambda}\left[\varphi''+\left(\frac{2}{r}+\frac{\nu'-\lambda'}{2}\right)\varphi'\right]+(\varepsilon^2e^{-\nu}-m_k^2)\varphi=0. \tag{7}$$

Here k = 1 inside the particle and k = 0 outside the particle.

It is obvious that the system (5)-(7) is incompatible with equation (3) because when $\lambda = 0$, the left side of (5) is equal to zero while the right side is negative for the values of the constants which have a physical meaning: $m_k^2 > 0$; $-m_0^2 < \lambda < m_0^2$.

We reduce the system of equations (5)-(7) to dimensionless form by making the substitutions $u=\frac{\sqrt{8\pi k}}{c^2}\varphi$, $x=\varepsilon r$, $\eta_k=\frac{m_k^2}{\varepsilon^2}$.

Then

$$e^{-\lambda}\left(\frac{1}{x^2}-\frac{\lambda'}{x}\right)-\frac{1}{x^2}=-e^{-\lambda}u'^2-(e^{-\nu}+\eta_k)u^2, \tag{8}$$

$$e^{-\lambda}\left(\frac{1}{x^2}+\frac{\nu'}{x}\right)-\frac{1}{x^2}=e^{-\lambda}u'^2+(e^{-\nu}-\eta_k)u^2, \tag{9}$$

$$e^{-\lambda}\left(u''+\frac{2}{x}u'+\frac{\nu'-\lambda'}{2}u'\right)+(e^{-\nu}-\eta_k)u=0. \tag{10}$$

The solution of system (8)-(10) in the neighborhood of zero can be investigated by representing the unknown functions in the form of a power series. Direct calculation gives us

$$u=a\left\{1+\frac{\eta_1-\beta}{6}x^2+\left[\frac{(\eta_1-\beta)^2}{120}+\frac{a^2(5\eta_1^2-6\eta_1\beta+4\beta^2)}{180}\right]x^4+\ldots\right\}, \tag{11}$$

$$e^{-\lambda}=1-\frac{a^2(\eta_1+\beta)}{3}x^2-\left[\frac{2a^2(\eta_1-\beta)(2\eta_1+\beta)}{45}+\frac{a^4(\eta_1-2\beta)\beta}{15}\right]x^4+\ldots, \tag{12}$$

$$e^{-\nu}=\beta\left\{\left[1+\frac{a^2(\eta_1-2\beta)}{3}x^2+(\eta_1-2\beta)a^2\times\right.\right.$$
$$\left.\left.\times\left[\frac{\eta_1-\beta}{30}+\frac{a^2(5\eta_1-7\beta)}{45}\right]x^4+\ldots\right\}. \tag{13}$$

Thus we have solutions limited at zero, with which (11) can be rewritten in the form

$$u = a\left[\frac{\sin(\sqrt{\beta-\eta_1}\,x)}{x} + a^2 f(x)\right], \tag{14}$$

where f(0) = 0.

It is obvious (ref. 10), as can be expected, that $\lambda(0)=0$; $\nu(0)<1$, and near zero the potential φ decreases faster than with an absence of gravitation, so that $\beta-\eta_1>1-\eta_1$.

At great distances from the center, symmetries λ and ν must asymptotically approach the solution of Schwartzfield: $e^{-\lambda}\approx 1-\frac{2\mu\varepsilon k}{c^2 x}\approx e^{-\nu}$. On the other hand, equation (8) can be rewritten in the form

$$e^{-\lambda} = 1 - \frac{1}{x}\int_0^x x^2\left(e^{-\lambda}u'^2 + e^{-\nu}u^2 + \eta_k u^2\right)dx.$$

From this we obtain the expression for mass

$$\mu = \frac{c^3}{2\varepsilon k}\int_0^\infty x^2\left(e^{-\lambda}u'^2 + e^{-\nu}u^2 + \eta_k u^2\right)dx. \tag{15}$$

We notice that in view of the rapid decrease of function n with an increase of the distance from the center, the upper boundary of the integral may be considered finite.

Figure 1 qualitatively shows the dependence of established functions $e^{-\lambda}$, $e^{-\nu}$ and φ, where the point of curvature of the potential φ (scale not maintained) corresponds to the radius of the particle.

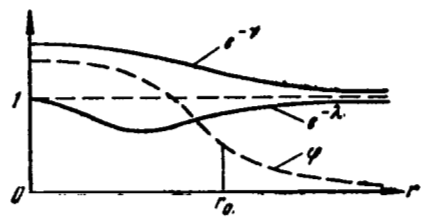

Figure 1

An analogous dependence of the metrics on the distance also takes place in the case of cubic nonlinearity, since in this case the limitedness and evenness of potential φ determines the limitedness of the metrics.

We estimate the corrections, determined by gravitation, for the concrete dimensions of the radius of particles r_0 and mass μ.

We can estimate the radius of the particle from relation (14), since the point of curvature corresponds to the value of the argument of the order of unity: $(\beta - \eta_1) x_0^2 \sim 1$.

The particle mass can be estimated by substitution in (15) of the upper boundary $x_0 = \varepsilon r_0$:

$$\mu \sim \frac{a^2 c^2 (\beta + \eta_1)}{2\varepsilon k} x_0^3.$$

from which

$$a^2 \sim \frac{2k\mu}{c^2 r_0} = \frac{r_g}{r_0}, \tag{16}$$

where r_g is the particle's gravitational radius. As is seen from (16), the gravitational correction will be smaller, the smaller the relation of the gravitational and true particle radii is. Thus, for $r_0 = 10^{-13}$ cm and $\mu = 10^{-27}$ g we obtain a disappearing, minute gravitational correction: $\alpha^2 \sim 10^{-42}$. However, the question of whether the stability of the particle is retained under the action of gravitation requires a special investigation.

The results we have obtained have a simple physical meaning: if the particle is "spread out" the density of its mass is small compared with the mass density of a hypothetical particle which is forced into the limits of the gravitational radius. The gravitational forces (as pointed out in all textbooks) are negligibly small compared with other forces which the nonlinear theory attempts to describe. However it is not entirely obvious that the energy of all elementary particles is concentrated within the limits of their classical radius, and for particles with a radius much less than the classical, the gravitational forces may play a substantial role.

In conclusion, I take this opportunity to express my gratitude to Professor Ya. P. Terletskiy for his interest and direction of the work

Department of Theoretical Physics.

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REFERENCES

1. Finkelstein, R., Levier, R., Ruderman, M. Phys. Rev., 83, 326-332, 1951.

2. Mitskevich, N. Zhurnal Eksperimental'noy i Teoreticheskoy Fiziki (ZhETF), 29, 354, 1955.

3. Finkelstein, R., Fronsdal, C., Klaus, D. Phys. Rev., 109, 1571, 1956.

4. Glasko, V. B., Leryust, F., Terletskiy, Ya. P., Shushurin, S. F. ZhETF, 35, 452, 1958.

5. Shushurin, S. F. Dissertation (Dissertatsiya). MOPI, 1961.

6. Rybakov, Yu. P. Vestnik Moskovskogo Universiteta (VMU), seriya fiziki, astronomii, No. 4, 27, 1962.

7. Shushurin, S. F. VMU, seriya fiziki, astronomii, No. 5, 1962.

8. DeBrogle, L. G. Phys. et Rad., 8, 225, 1927.

9. Shirokov, M. F. VMU, seriya mat., mekh., astron., fiz., khimii, No. 4, 1957.

10. Landau, L. D., Lifshits, Ye. M. Field Theory (Teoriya polya). Fizmatgiz, Moscow, 1959.

11. Oing, J. General Theory of Relativity (Obshchaya teoriya otnositel'nosti). IL, Moscow, 1963.

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